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Debt, Dividend Policy, Taxes, Inflation and Market Valuation

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I. Introduction

IN THIS CONTRIBUTION, I propose to go once more over two "core issues" of corporate finance—how do leverage and dividend policy affect market valuation? These are issues that were supposed to have been settled in the two contributions that students often refer to as MoMi (Modigliani and Miller [1958]) and MiMo (Miller and Modigliani [1961]). The basic message in these papers, was that with rational investors, well functioning markets, and no taxes (or at most, only of a certain type), financial policy does not matter!

But that conclusion did not carry over to a world with taxes, which have been, and still are, a continuing source of trouble. After a false start in MoMi, the MM "Correction" [1963], on the assumption that (i) all corporate returns are taxed equally at the personal level, and (ii) the tax saving from the use of debt can be regarded as a perpetual riskless flow, concluded that each dollar of debt in the capital structure should add to the market value of the firm at a rate equal to the corporate income tax.

The gain from leverage results, derived from the demand-side analysis, in turn created immediate problems. If debt is valuable, why should firms not be financed as nearly as possible by debt alone, with the optimum leverage representing a corner solution? This implication was disturbing because it was both counterfactual and against common sense. Accordingly, it has given rise to a good deal of work focussing on the supply side. This analysis has uncovered and analyzed four major ways in which leverage could unfavorably affect the market value of the firm: (i) the most obvious and traditional is through bankruptcy costs which reduce the expected flow to all concerned (numerous references, beginning with 1967, are provided in Williamson [1981], p. 18); (ii) a second is through agency costs, resulting from the arrangements needed to protect the creditors (Jensen

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and Meckling [1976]; Chen and Kim [1979]); (iii) the third, and most sophisticated way, is what might be called moral hazard or foregone valuable opportunities (Myers [1977]) which would be particularly relevant for firms with true growth opportunities. A fourth and most recent argument for an interior solution relies on the consideration, first developed by Brennan and Schwartz [1978], that debt is valuable in so far as it serves to shelter income from taxes, though at a cost. As debt rises, there is a growing probability of income falling below a threshold level where the shelter can not be used. This argument has since been generalized by taking into account the availability of shelters other than debt and, in particular, tax credits (DeAngelo and Masulis [1980]; Cordes and Sheffrin [1981]).

In the midst of the efforts to explain why the supply of debt would be limited, even if leverage commanded a positive price, Miller [1977] took a radical turn on the demand side. He relies on the consideration that, contrary to the MM assumption, the personal taxation of equity returns is typically lower than that of interest—and the differential is larger the higher the tax bracket. As first shown by Farrar and Selwyn [1967], this implies that the *personal* value of leverage to any investor is a decreasing function of his income tax rate, becoming negative once that rate exceeds the corporate income tax rate by a factor depending on the capital gain rate. Now, Miller argues, the *market* valuation of leverage must reflect the personal tax of the *marginal* holder of corporate debt, and, given the availability of tax exempt securities, that marginal rate must rise continuously as the quantity of corporate debt outstanding rises. If the supply of debt is infinitely elastic and the top personal tax rate exceeds the corporate rate, as he assumes, then, in equilibrium the market value of leverage must be zero.

Ever since the appearance of Miller's 1977 paper, I have been skeptical of his conclusions. First, I found unconvincing his off-hand dismissal of factors limiting the supply of debt; if the supply of debt has costs, then the intersection of demand and supply can only come at a point where debt is valuable at the margin, and therefore leverage is a serious issue of financial policy. Second, I felt uneasy that his argument rested on tax exempt securities whose rate was taken as exogenously given, an approach that has generally been accepted by those who have since made use of his model, favorably or critically, (Lewellen, Stanley, Lease and Schlarbaum [1978]; Taggart [1980]). I suspected that the validity of his argument could not depend on the existence of tax exempt debt. Furthermore, his model implied a counterfactual coincidence between the ratio of tax exempt to fully taxed interest on the one hand, and the corporate tax rate on the other (Gordon and Malkiel [1981]; Skelton [1980]). But, in the process of validating these suspicions—a task which has since been at least partly accomplished by others (e.g., DeAngelo and Masulis [1980])-I discovered serious difficulties with Miller's framework, because of its tendency to lead to unstable corner solutions.

It soon became apparent that these counterfactual implications came from failure to properly take into account the role of diversification in a world of uncertainty and risk aversion. I was, thus, led to pursue the problem in the mean-variance framework, along lines first successfully considered in the pioneering contribution of Brennan [1970], and later extended to the analysis of individual portfolios by Elton and Gruber [1978] and Auerbach and King [1981].

In what follows, I propose to show that this framework can provide clear answers to a good number of long standing issues on the effect of alternative financial policies on market valuation and portfolio composition in the presence of various taxes and of (steady) inflation. Furthermore, the results, which coincide with MM in limiting cases, are intuitively appealing, and appear broadly consistent with empirical evidence, though with one important exception to be discussed presently, pertaining to the effects of inflation.

Because of space limitations, this paper concentrates on the demand side, taking the supply as largely given, though the concluding section outlines the integration of the demand with the supply side.

Some of the major conclusions can be summarized as follows:

1) We can expect, with great confidence, leverage to be valuable, but the value could be modest if, as one might expect, the market regards the tax saving flow as subject to risk, like the underlying profit stream, rather than as a sure perpetuity, as assumed in MM.

2) Inflation should increase the value of leverage and, through that route as well as others, also increase the *price-earnings ratio* (though not necessarily the *level* of stock prices).

3) The payment of dividends should, unequivocally, tend to reduce market value, but the effect could, again, be modest if the tax consequences are capitalized at a risky rather than at the sure rate.

4) Differential rates of taxation between investors as well as between sources of return, will result in clientele effects. That is, people having the same risk tolerance will have different portfolio compositions. Moving from the lowest brackets to the highest, one should find a steady rise in the share of the portfolio invested in stocks with low dividend yield (which generally accompany high true growth) and with relatively low betas, due in particular, to low leverage. However, the differences to be expected in the portfolio composition of a high versus a low tax bracket appear to be modest in contrast to Miller's corner type solutions.

5) Inflation can be expected to increase this polarization of portfolios.

II. Derivation of Market Equilibrium

1. *Individual demand*

In the spirit of the mean-variance model, we confine our attention largely to the market for equities and debt. We begin by deriving the demand for stocks and debt by individual investors from the maximization of the utility function, whose only arguments are portfolio mean and variance. We rely on the following notation: τ_c = corporate tax rate; τ_g = capital gain rate; τ_p = personal income tax rate; $\theta_x = 1 - \tau_x$; $x = c, g, p$; μ = cash flow (EBIT), $\mu^* = \theta_c\mu$; $[M]$ = variance-covariance matrix of tax adjusted cash flows, μ^*; S = market value of equity; D = net corporate debt; $V = S + D$ = market value of firm; Δ = dividend payment; R = nominal rate of interest; p = rate of inflation; $r = R - p$ = real rate of interest; $r_p = \theta_p R - p$ = real interest rate after personal taxes; $r_c = \theta_c R - p$. Finally, a letter superscript will characterize an investor, a subscript, a firm; the superscript ~ denotes a random variable; and a bold letter stands for a column vector.

The expected return to the mth investor, y_i^m, from holding a fraction, n_i^m, of the equity of firm i, net of all corporate and personal taxes, and allowing for a fully anticipated, constant rate of inflation, p, can now be written as:

$$\begin{aligned} y_i^m &= n_i^m\{[\mu_i - \tau_c(\mu_i - RD_i) - RD_i + pD_i - \Delta_i]\theta_g^m + \Delta_i\theta_p^m\} \\ &= n_i^m[(\mu_i^* - r_cD_i)\theta_g^m + \Delta_i(\theta_p^m - \theta_g^m)] \end{aligned} \qquad \text{II.1}$$

Making use of II.1 and the standard budget equation, we can express the total expected portfolio return to the mth individual, having wealth w^m, as:

$$y^m = (w^m - \textstyle\sum_i n_i^m S_i)r_p^m + \sum_i n_i^m[(\mu_i^* - r_cD_i)\theta_g^m + \Delta_i(\theta_p^m - \theta_g^m)] \qquad \text{II.2}$$

where r_p^m is the after-personal-tax real rate, $\theta_p^m R - p$.

We will pursue first the implications of the "traditional" MM assumption that the flows associated with debt are both permanent and riskless. If we extend this assumption also to dividends, the only stochastic component of returns in II.2 is μ_i^*, and the variance of the portfolio return for the mth individual takes the form:

$$(\sigma_y m)^2 = E(\tilde{y}^m - y^m)^2 = (\theta_g^m)^2 \textstyle\sum_i \sum_j n_i^m \mu_{ij}^* n_j^m \qquad \text{II.3}$$

where μ_{ij}^* is the covariance of $\tilde{\mu}_i^*$ with $\tilde{\mu}_j^*$.

Maximization of a utility function of the form:

$$u^m = u^m[y^m, (\sigma_y m)^2],\ u_1^m > 0,\ u_2^m < 0, \quad \text{and} \quad u_2^m/u_1^m = -\gamma^m/2$$

with respect to n_i^m, $i = 1, 2, \cdots, N$, subject to the budget constraint, leads to a system of first order conditions that can be expressed in vector notation as:

$$(\boldsymbol{\mu}^* - r_c\mathbf{D})\theta_g^m - \mathbf{S}r_p^m - (\theta_g^m - \theta_p^m)\boldsymbol{\Delta} = \gamma^m(\theta_g^m)^2[M]\mathbf{n}^m \qquad \text{II.4}$$

where $\boldsymbol{\mu}^*$ is a column vector of the μ_i^*'s, and similarly for $\mathbf{D}$, $\mathbf{S}$, etc., and $[M]$ is the variance-covariance matrix of the μ_i^*'s.

2. *Market equilibrium*

As shown by Brennan [1970], one can, from these conditions, obtain a general equilibrium solution for the price of all "firms" (or risk classes). To this end, multiply both sides of II.4 by:

$$\frac{\Lambda}{\gamma^m(\theta_g^m)^2}, \quad \text{where} \quad \Lambda = 1/\textstyle\sum_m \dfrac{1}{\gamma^m(\theta_g^m)^2}$$

and then sum over all individuals. Taking into account the fact that:

$$\textstyle\sum_m n_i^m = 1,\ \forall\ i$$

the result of this summation can be written as:

$$(\boldsymbol{\mu}^* - r_c\mathbf{D})\theta_g - \mathbf{S}r_p - \boldsymbol{\Delta}(\theta_g - \theta_p) = \Lambda[M]\mathbf{1} \qquad \text{II.5}$$

Here:

$$\theta_g = \textstyle\sum_m \theta_g^m \times \dfrac{\Lambda}{\gamma^m(\theta_g^m)^2} \qquad \text{II.5A}$$

is an average of the θ_g^m, weighted by $\frac{1}{\gamma^m(\theta_g^m)^2}$, Λ being the inverse of the sum of the weights [and also $1/M$ the harmonic average of the quantities $\gamma^m(\theta_g^m)^2$]. Similarly, $r_p = \theta_p R - p$, with θ_p the same weighted average of the θ_p^m.

The market value of each firm's equity is obtained by solving II.5 for S. To obtain the market value of the firm as a whole, $V_i = S_i + D_i$, we can add and subtract $\mathbf{D}r_p$ on the left hand side of II.5, rearrange terms, and solve for V to obtain:

$$\mathbf{V} = \frac{1}{r_p}[\boldsymbol{\mu}^*\theta_g - \boldsymbol{\Delta}(\theta_g - \theta_p) - \Lambda[M]\mathbf{1}] + l\mathbf{D} \qquad \text{II.6}$$

where the ith row of $[M]\mathbf{1}$ is $\operatorname{cov}(\mu_i \mu)$, and:

$$l = \left(1 - \frac{r_c}{r_p}\theta_g\right) \qquad \text{II.7}$$

is the "value of leverage" to be discussed presently.

Equations II.5 and II.6 can also be usefully restated in terms of risk premia and CAPM β's. To this end we sum the equations II.6 over all firms to obtain:

$$V - lD + \frac{\theta_g - \theta_p}{r_p}\Delta \equiv V^* = \frac{\mu^*\theta_g - \Lambda \operatorname{var}(\tilde{\mu}^*)}{r_p} \qquad \text{II.8}$$

Here, μ^*, Δ and V are the aggregate cash flow, dividend and total value respectively, $\operatorname{var}(\tilde{\mu}^*)$ is the variance of overall market returns, while V^* denotes that part of market value that is due to the risky return, and excludes the portion, if any, due to the capitalization of the government contribution through tax shields and treatment of dividends. Thus, V^* corresponds to the MM notion of the "value of an unlevered stream". II.8 implies:

$$\frac{\Lambda \operatorname{var}(\tilde{\mu}^*)}{V^*} = \frac{\mu^*\theta_g}{V^*} - r_p \equiv \pi \qquad \text{II.9}$$

and $(\mu^*/V^*)\theta_g$ is the after (average) tax rate of return on the unlevered market, in the absence of dividends. The expression after the first equality in II.9 can therefore be labeled the "tax adjusted excess return"—the difference between the market return and the interest rate, after both are adjusted by the weighted average tax rate defined by II.5A. II.9 tells us that this difference, or risk premium, which is denoted by π, is proportional to the variance of market returns adjusted for taxes (implicit in Λ), a generalization of the familiar CAPM result.

Using the above definition of π, one can express total market value as:

$$V = \frac{\mu^*\theta_g}{r_p + \pi} + lD - \frac{\theta_g - \theta_p}{r_p}\Delta \qquad \text{II.10}$$

Expressions analogous to II.10 can also be derived for individual stocks by defining:

$$V_i^* = V_i - lD_i + \Delta_i(\theta_g - \theta_p)/r_p \qquad \text{II.11a}$$

$$\beta_i = \operatorname{cov}\left(\frac{\mu_i^*}{V_i^*}, \frac{\mu^*}{V^*}\right) \qquad \text{II.11b}$$

Using II.11b and II.9, $\Lambda[M]1$ in the right hand side of II.6 can be replaced by $(\mathbf{V}\boldsymbol{\beta})\pi$, and the equation can then be solved for V. For the ith firm, one obtains:

$$V_i = \frac{\mu_i^* \theta_g}{r_p + \beta_i \pi} + lD_i - \Delta_i \frac{\theta_g - \theta_p}{r_p} \qquad \text{II.12}$$

Subtracting the debt from both sides of II.12 yields an expression for the value of equity:

$$\begin{aligned} S_i &= \frac{\mu_i^* \theta_g}{r_p + \beta_i \pi} - (1 - l)D_i - \Delta_i \frac{\theta_g - \theta_p}{r_p} \\ &= \frac{(\mu_i^* - r_c D_i)\theta_g}{r_p + \beta_i^* \pi} - \Delta_i \frac{\theta_g - \theta_p}{r_p} \end{aligned} \qquad \text{II.13}$$

where

$$\beta_i^* = \beta_i \left[1 + d_i' \frac{r_c \theta_g}{r_p} \right] \quad \text{and} \quad d_i' = D_i \Big/ \left[S_i + \Delta_i \frac{\theta_g - \theta_p}{r_p} \right]. \qquad \text{II.13a}$$

The second equality is a rearrangement of the first for the purpose of expressing S in terms of stockholders' profits.

II.12 and II.13 are seen to be consistent with well known MM "Correction" formulae for the special case $\theta_g^m = \theta_p^m$ and no inflation (Modigliani and Miller [1963], equations (3) and (6) respectively), but they enable us to analyze the effect of financial policies under more general conditions, including inflation.

3. *The effect of financial policies in the absence of inflation*

i) *The value of leverage*

Remembering the definition of r_c and r_p, the value of leverage l, given in II.7, can be written in the following form, which separates out the effect of inflation:

$$l = q + (q - \tau_g)\frac{p}{r_p}; \quad q = 1 - \theta_c \theta_g / \theta_p \qquad \text{II.14}$$

For $p = 0$, l is seen to reduce to q, an expression made familiar by Farrar and Selwyn [1967]. It is consistent with the *MM* formulae and also with Miller's $l = 0$, if $\theta_g \theta_c = \theta_p$. But, according to our analysis, there is no reason why this equality should hold (at least for the U.S.). Indeed, with θ_c around 0.5, the value of $\theta_c \theta_g$ can not be appreciably in excess of 0.45, while the value of θ_p must, assuredly, be significantly higher, most likely around ⅔. This assessment of θ_p is supported by two considerations.

First, according to Miller, what insures the equality of θ_p and $\theta_c \theta_g$ is that θ_p represents the tax factor of the marginal holder of levered stock (i.e., the holder for whom leverage is least advantageous). Accordingly, θ_p decreases with the supply of leverage. An infinitely elastic supply together with sufficiently high tax brackets then insures that, in equilibrium, q must be zero. But once we take into account the benefits of diversification, the value of leverage, l, is *independent* of its supply and depends instead on the (weighted) *average* tax factors θ_p, and θ_g, and on θ_c.

The average value of τ_p^m, the personal tax rate, can probably be put at somewhat less than 0.4 (for some empirical evidence see, e.g. Lewellen, Stanley, Lease and Schlarbaum [1978]; Feldstein and Summers [1979]; Kim, Lewellen and McConnell [1979]). But the value of τ_p relevant in II.14 should be appreciably less than this average. In the first place, one should allow for tax exempt investors. Secondly, as long as there is a substantial market for tax exempt securities, the "effective" rate for any investor should be either his actual rate or $(1 - i/R)$, where i is the (nominal) rate on tax exempt securities, whichever is lower. For, clearly, regardless of his income, an investor always has the opportunity to invest in tax exempt securities yielding i, which is equivalent to investing in conventional debt at an "effective" rate, $\tau_p = 1 - (i/R)$. Thus, as long as i/R is larger than $\theta_c\theta_g$, corporate leverage is valuable to all investors, regardless of income. Now, at least for long maturities, i/R has consistently been well above one half. It has, in fact, tended to stay around 0.7 and uniformly above 0.6 (Mussa and Kormendi [1973]; Skelton [1980]). This suggests, therefore, that the average of the effective values of θ_p^m can be safely placed at no less than ⅔. If so, in the absence of inflation, l could be expected to be positive with a value around ⅓, compared with *MM*'s τ_c or 0.5. Furthermore (as can be verified from II.12), if one can suppose that an expansion of interest payments is necessary and sufficient to enable the firm to cut dividends by an equal amount, then the (marginal) value of leverage is the full *MM*'s τ_c (Brennan [1970]; Litzenberger [1980]).

An alternative way of assessing the value of leverage is to ask how much it can contribute to market value in relative terms. We find:

$$\frac{\partial V_\iota}{V_\iota} = \frac{q}{\theta_g}(r_p + \beta_\iota \pi)\,\partial d \qquad \text{II.15}$$

where $d = D_\iota/\mu_\iota^*$ and the derivative is evaluated at $D_i, \Delta_\iota = 0$. With q/θ_g estimated at below .4, the value of this expression for the "average" β of 1 can be put at between .02 and .03. Thus, a rise in d from 0 to as much as 10, and even neglecting any "cost" of leverage, would generate a rise in value of around 25 per cent—not negligible but not very large either.[1]

ii) *Valuation and dividend policy*

Since θ_g can be taken as larger than θ_p, at least for the U.S., from II.12 we can infer that the payment of dividends must unequivocally reduce the market value of the firm, at least within the set of cost-benefits explicitly modeled. This conclusion agrees with Brennan's result [1970] and with a widely held view, though it is hard to reconcile with prevailing payout policies.

If the market regarded the flow of extra taxes associated with dividends as fixed and perpetual as assumed in II.10, then the unfavorable effect of dividends on value would appear to be hefty. For instance, with the usual assumptions about the tax parameters, and assuming further that r_p is of the order of ½ π, one finds that an increase in the payout from zero to 10 percent would have the effect of reducing market value by some 8 percent, or even more for a high β stock. But

[1] II.15 suggests that the effect could be substantially larger, for high β stocks. But this implication is questionable—see section II.5 below.

this estimate is surrounded by considerable uncertainty. On the one hand, it might be argued that the value of τ_p applicable to dividends might be higher than that relevant to the leverage effect. But, on the other hand, Miller and Scholes [1981] have attempted to show (even if not entirely convincingly) that the relevant $\tau_p - \tau_g$ might be small or negative! Finally, the estimate would also be reduced if, as seems plausible, the market takes as given the payout policy rather than the dividend itself (see 5 below).

4. *The impact of inflation on valuation*

As is apparent from II.12, 13, and 14, inflation has an effect on valuation through, and only through, the real after tax rates r_p and r_c. Hence its impact depends critically on how these rates, particularly r_p, respond to inflation.

i) *Effect on interest rates*

Unfortunately, as is well known, the interest rate cannot be derived from the CAPM framework alone. We propose to handle this difficulty by relying on an approach which is appealing, at least if we are concerned with long run implications. It consists in adding an equation expressing the requirement that, in long run equilibrium, the market value of corporate capital must coincide with the reproduction cost of that capital (or, that Tobin's q should be one). Thus:

$$V = K$$

where K is the reproduction cost.[2]

Substituting K for V in II.8, we can solve that equation for the real rate, r. For present purposes, however, it turns out to be more convenient to solve for the real after tax rate, r_p. This yields:

$$r_p = \theta_p r - \tau_p p = \left[p\theta_g \frac{\theta_p - \theta_c}{\theta_p} d^* + \rho\theta_c\theta_g - \delta^*(\theta_g - \theta_p) - \Lambda\theta_c^2 \operatorname{Var}(\rho) K \right] / (1 - qd^*) \qquad \text{II.16}$$

where $\rho = \mu/K$, $d^* = D/K$ and $\delta^* = \Delta/K$.

As one should expect, r_p rises with the net-of-tax return from assets and declines with the risk premium (the last term in the numerator). But, the effect of lowering θ_c or θ_g (raising taxes) is uncertain since it lowers the after tax return but it also lowers its variance (provided losses can be fully recouped).

The effects of inflation are captured primarily through the first term. Because θ_p exceeds θ_c, inflation should have a positive effect on r_p (through leverage). But, on the basis of our estimate of the tax coefficient, this effect is moderate: the tax factor comes to just over 0.2, so that with, say, a 25 percent average ratio of debt to total capital, 10 percent inflation would raise r_p by some 50 basis points.

The remaining terms do not involve p explicitly, but could nonetheless be systematically affected by inflation. In particular, even supposing that p is neutral

[2] As a result of tax effects, the market value may be taken as proportional rather than equal to reproduction cost.

with respect to K, the after tax return, $\rho\theta_c$, is likely to be systematically reduced through the taxation of paper profits from inflation. Suppose, for the moment, that the leverage and $\rho\theta_c$ effects roughly cancel. Then, II.16 says that inflation should leave unchanged the *after* (average) *tax*, *real* personal *rate* of return ($R\theta_p - p$). Thus it confirms the conclusion already reached by others (e.g., Feldstein [1976]; Summers [1981a]) that the nominal rate should *not rise* by p as stated by Fisher's Law, but rather p/θ_p (or, for the U.S., some one-and-a-half times as much as inflation), a proposition that may be labelled "Super Fisher's Law." Of course, even if r_p is inflation invariant, the after tax real rate will fall for those in above-average brackets, including corporations, and rise for low ones. In addition, the leverage and $\rho\theta_c$ effect need not offset each other precisely and, accordingly, the nominal and real rates might rise somewhat more or less than called for by the Super Fisher's Law. However, with significant inflation, one should definitely expect a nonnegligible rise in the (before-tax) real rate.

ii) Inflation and the value of leverage

To see what these results imply for the impact of inflation on the value of leverage, one can turn to II.14. Suppose inflation left roughly unchanged the after tax rate, r_p, as suggested by the result of the previous section. In this case, l can be seen to be a linear function of p. Furthermore, the coefficient of p/r_p is close to that of q, since τ_g is 0.1 or even less. Now, r_p is presumably a rather small number, say 2 to 3 percent. That implies a quite large response of l to inflation—a rise in p from zero to no more than 4 percent would be sufficient to double l! These results reflect the fact that, even if inflation does not increase the advantage of personal borrowing (r_p is constant), it will reduce r_c increasing the gains from corporate debt, and the gain is capitalized at a low rate. Even more dramatic effects should result if r_p declines—for example, because the *real* rate is constant.

iii) *Inflation and the stock market*

III.13 shows that the response of equity values to a (permanent) rise in the anticipated rate of inflation should reflect the effect of that rise on three components of S: (i) μ^*; (ii) l; and (iii) r_p. The effect on the first component unequivocally tends to lower S. But it might be more than offset by the effect on the remaining two components, which we have shown to push in an upward direction. This is true even if the higher tax burden, through the taxation of paper profits, has exceeded the lower burden due to the treatment of interest, and other effects, as has been maintained, e.g. by Feldstein and Summers [1979] (though the issue is by no means settled; see, e.g. Briden [1981]). For this development should have been offset by the fall in r_p, at least through the 70s, whether or not this fall is related to the higher tax burden claimed by Feldstein and Summers.

But whether S should have risen or not, one inference that can be clearly drawn from II.13 is that the price-earnings ratio (P/E) should have risen, and very substantially, during the rising inflation—for this effect depends only on components (ii) and (iii), both of which push unequivocally and strongly in an upward direction. Furthermore, the rise in P/E should be greater the higher the leverage.

To conclude, the model clearly implies that inflation should exert a pronounced upward thrust on the value of leverage and on P/E. However, there are grounds for discounting the implied large quantitative effects, especially those associated with possible near zero or even negative values of r_p, as a result of inflation. Actually, such values must be ruled out if r_p is to be interpreted as the "consol" rate. To be sure, a negative real rate is perfectly possible, but *not* if there exist *perpetual* positive real streams of returns (e.g., via tax saving).

This observation points to one important limitation of our analysis of the value of leverage and dividends, namely that the results are strictly applicable only when the relevant variables are expected to be *indefinitely* maintained and *certain.* This limitation must be kept in mind in interpreting all results, but particularly those associated with nominal quantities, such as R and p. There is no basis for the expectation that a high level of p and R would be preserved forever. Furthermore, if inflation is not neutral (notably because of nominal institutions or illusions), it is very doubtful that investors would, or should, act as though any given constellation of R, p, and institutions would last forever, especially if it gives rise to unusual advantages.

Another factor that could be counted on to reduce the magnitude of the response to inflation is the stochastic rather than deterministic nature of tax saving flows associated with alternative financial policies, to which we now turn.

5. *The implications of uncertain tax savings and dividends*

The implications of the market regarding the future stream of interest and dividend payments and the associated tax consequences as stochastic, rather than as certain, perpetual streams, depend on the stochastic properties of the expected streams. For the sake of illustration, we shall pursue the implications of the market taking as given, and nonstochastic, the debt-equity and payout policy, d_i and δ_i, rather than the interest and dividend flows. (This formulation has been suggested, with particular reference to debt and capital budgeting, by Miles and Ezzell [1980].)

The total return from firm i can now be written as:

$$\tilde{y}_i^m = \tilde{\mu}_i^*[\theta_g^m + (r_p^m - r_c\theta_g^m)d_i - \delta_i(\theta_g^m - \theta_p^m)]$$

where $d = D/\mu^*$, $\delta = \Delta/\mu^*$. Since equity holders are presumed to absorb all the risk, the variance associated with the equity must be $\text{var}(\tilde{y}_i)$.

By repeating the steps leading from II.4 to II.12, one obtains the following reformulation of equation II.12:

$$V_i = \mu_i^*[\theta_g + d_i(r_p - r_c\theta_g) - \delta_i(\theta_g - \theta_p)]/(r_p + \bar{\beta}_i\pi) \qquad \text{II.17}$$

Here the tax parameters are again averages of the individual investor's rates weighted by his shares of the market, $\bar{\beta}_i$ is a similarly weighted average of the β_i's of the individual portfolios, and π is the spread between the overall after tax rate of return on equity and the average after tax interest rate.[3]

[3] This proposition holds on the plausible assumption that $n_i^m\beta_i^m$ is not appreciably correlated with π^m across investors.

From II.17, the value of leverage becomes:

$$l^* = \frac{dV_l}{dD_l} = \frac{lr_p}{\beta_l \pi + r_p} \quad \text{or} \quad \frac{1}{V}\frac{dV}{dd^*} = \frac{qr_p}{\theta_g}$$

This suggests l^* can be put at roughly one-third the value of l implied by II.14, or at a bit above .10, and similarly for the relative change in value. The effect of dividend policy on value is also reduced by a factor of ⅔, to a fairly modest effect, suggesting that in setting dividend policy, tax considerations could well be swamped by other factors.

In summary, there can be little doubt that the estimate of l of ⅓ obtained under the assumption of perpetual, sure consequences suffers from serious upward bias, but there is little basis for pinpointing its magnitude. Even .10 need not provide a lower bound. It is interesting nonetheless, that the latter figure is not inconsistent with the recent estimates by Masulis [1981] relying on the ingenious technique of measuring the impact of various types of exchange offers on the price of common stock.[4]

It is much harder to assess the consistency with the evidence of our results for dividends, mainly because at present the evidence itself is so controversial. Some authors find no evidence in support of the hypothesis that dividends are valued significantly below other corporate returns,[5] while others report strong evidence consistent with that hypothesis, especially when allowing for the valuation to vary with the size of dividend yield, as implied by tax clientele effects.[6] In the latest application of the latter approach, Auerbach [1981] has estimated the effect of dividends on returns over a 15 year period for each of a large sample of firms. His average coefficient, .22, compared favorably with the value implied by II.17, $(\theta_g - \theta_p)/\theta_g$, or around .25.

At the same time, the implications of the model about the effects of inflation seem to be grossly inconsistent with the empirical evidence: (i) the after tax real rate has declined markedly, and even the real rate has hardly kept up during most of the last 15 years (for an analysis of earlier experiences, see Summers [1981a]); (ii) equity values, and most particularly price-earnings ratios, have declined dramatically, whereas they should have risen appreciably, especially in light of the decline in r_p; (iii) levered firms, instead of appreciating, have tended to lose value, at least according to one recent study;[7] and (iv) there is little

[4] While his "preferred" estimate is below .10, it should be remembered that it reflects also effects associated with the "cost of leverage" (see conclusions). In fact, if there is an interior optimum debt and the exchange were aimed at that optimum, the effect of the exchange on value would be appreciably smaller than l.

[5] See, e.g., Black and Scholes [1974], Gordon and Bradford [1980], and Miller and Scholes [1981].

[6] Elton and Gruber [1970]; Litzenberger and Ramaswamy [1980]. The Elton and Gruber results have been severely criticized and called in question by Kalay [1977], but it is not clear that his criticism applies to the other studies. It may be noted in this connection that even the results of Miller and Scholes, when they classify firms by dividend yield, are strikingly supportive of the hypothesis of a dividend effect, despite the authors' attempt at discrediting them by calling attention to outliers in one of the nine cells of the table.

[7] The study, carried out by myself and Richard Cohn, is still unpublished, but a short description is provided in Modigliani and Cohn [1982]. However, quite different results have been reported recently by Summers [1981b], using a somewhat different approach.

evidence of any significant increase in leverage, even though its profitability should have gone up.[8]

We suspect that this total failure can be attributed to the assumption of rational behavior, whereas in the presence of inflation, at least of a moderate size, the market may suffer from inflation illusion of the type hypothesized by Modigliani and Cohn [1979]. It can be verified that dropping the term p from II.1, and replacing R_p for r_p in equations like II.10 and II.12, as suggested by that hypothesis, would go a long way toward explaining the contradictions highlighted above.

III. Individual Portfolio Composition

1. *Taxes and portfolio allocation—basic characteristics*

It is well known that, in the absence of taxes, and with uniform assessments, the mean-variance approach implies that the optimum portfolio of every investor will hold the same fraction of every firm's equity.

As has been shown by a number of authors (Black [1971, 1973], Elton and Gruber [1978], Gordon and Malkiel [1981], Auerbach and King [1981]) this proposition no longer holds when investors are differently taxed. This conclusion can be verified by solving the equilibrium condition II.5 for S and substituting in the first order condition II.4. This yields, after some rearrangement:

$$(\mu^* - r_c\mathbf{D})\left(\theta_g^m - \frac{r_p^m}{r_p}\theta_g\right) - \mathbf{\Delta}\left[(\theta_g^m - \theta_p^m) - \frac{r_p^m}{r_p}(\theta_g - \theta_p)\right] + \frac{r_p^m}{r_p}\Lambda[M]\mathbf{1} = \gamma^m(\theta_g^m)^2[M]\mathbf{n}^m \qquad \text{III.1}$$

The system can be solved for $\mathbf{n}^m$ (assuming $[M]$ nonsingular) and the solution can be cast in the form:

$$\mathbf{n}^m = \frac{\Lambda}{\gamma^m}\left\{T_1^m\mathbf{1} + \frac{1}{\Lambda}[M]^{-1}[(\mu^* - r_c\mathbf{D})T_2^m - T_3^m\mathbf{\Delta}]\right\} \qquad \text{III.2}$$

where

$$T_1^m = \frac{r_p^m/r_p}{(\theta_g^m)^2} \qquad \text{III.2a}$$

$$T_2^m = \frac{\theta_g^m - (r_p^m/r_p)\theta_g}{(\theta_g^m)^2} \qquad \text{III.2b}$$

$$T_3^m = [(\theta_g^m - \theta_p^m) - r_p^m/r_p(\theta_g - \theta_p)]/(\theta_g^m)^2 \qquad \text{III.2c}$$

[8] The evidence occasionally offered to support the proposition that leverage has increased dramatically, is uniformly based on measures which are irrelevant in the presence of inflation, such as using book value in the denominator or relying on market value, whose relevance is, at least, questionable in view of its dramatic decline (cf., for instance, Gordon and Malkiel [1981, Table 1]. If one relies on more suitable measures of the denominator, such as reproduction costs of assets, the association with inflation disappears (Gordon and Malkiel, ibid.), and the same tends to happen using a variable like *EBIT*.

In the absence of personal taxes, or, more generally, when everybody is taxed equally, III.2 implies the standard result, $n_i^m = \Lambda/\gamma^m$ for all i; every investor holds a share of the market, inversely related to the investor's risk aversion γ^m.

Next, if individuals were taxed differently, but (i) all property income was taxed at the *same* rate, or $\theta_g^m = \theta_p^m$ (as assumed implicitly in the MM "Correction" [1963]), and (ii) there was no inflation (or only real interest was taxed), then, since under those conditions $\theta_g = \theta_p$, and $r_p^m/r_p = \theta_p^m/\theta_p$, we find $T_2 = T_3 = 0$, and

$$n_i^m = \frac{\Lambda}{\gamma^m(\theta^m)^2}\ \forall\ i$$

Thus, each portfolio consists again of a share of the market, but that share now depends also on the investor's tax bracket, θ^m, relative to the average θ (implicit in Λ). Since θ^m is a decreasing function of the tax rate, for given risk preference, n^m is an increasing function of the investor's tax bracket. The reason, basically, is that a higher tax rate implies an equal decline in the (expected) return from equity and from debt, but it also implies a decline in the variance of the after tax outcome from equity, which makes equity relatively more attractive.

Finally, if, and only if, both individuals and sources of income are taxed differently, investors will hold different shares of any given firm depending on both their tax rates and their risk aversion. However, if any two investors, say, m and m', are subject to identical tax rates, then T_1, T_2, T_3 of III.2 will have the same value, and therefore: $n_i^m/n_i^{m'} = \gamma^{m'}/\gamma^m$, $\forall\ i$. Thus, it remains true that every investor could, in principle, secure his optimum portfolio by combining positive or negative debt with a share of a single appropriate "tax fund," which held shares in the relative quantities appropriate to his tax bracket, as given by III.2.

2. *Firms' characteristics and portfolio composition*

To examine how the portfolio composition of funds suited for different tax brackets should be expected to respond to various characteristics of the firm, it is convenient to consider first the special case where $[M]$ is diagonal. In this case, from III.2, the equation prescribing the holdings of the ith stock for a fund catering to investors with any given set of rates, say $\hat{\theta}_g$, $\hat{\theta}_p$, can be written as:

$$n_i = k[T_1 + x_i(T_2 - \delta_i T_3)] \qquad \text{III.3}$$

Here, T_1, T_2, and T_3, are the same as defined in III.2, but with θ_p^m, θ_g^m, r_p^m, replaced by $\hat{\theta}_p$, $\hat{\theta}_g$, $\hat{r}_p$; δ is the payout ratio and k is a proportionality factor that depends on the size of the fund. Finally:

$$x_i = \frac{(\mu_i^* - r_c D_i)}{\Lambda \mu_{ii}} = \frac{(\mu_i^* - r_c D_i)}{(\beta_i^* \pi S_i)} = \left(1\, pl \frac{r_p}{\beta_i^* \pi}\right)(\theta_g)^{-1} \qquad \text{III.3a}$$

where β_i^* is the β of the rate of return to equity, defined in II.13a. The second equality in III.3a follows from II.9 and the fact that, with a diagonal matrix, $\text{var}(\tilde{\mu}_i) = \text{cov}(\tilde{\mu}_i \tilde{\mu})$, while the last follows from II.13.

It is apparent from III.2 that both T_2 and T_3 are zero for the "average" tax fund which, therefore, holds the market, and that they are increasing functions of the tax rate, τ_p, as long as $d\tau_g/d\tau_p$ is well below unity, as is the case for the U.S. It then follows from III.3 that for a fund that caters to higher tax brackets, the relative share held of any stock will be higher the higher that stock's x (cf. Auerbach and King [1981]), and the lower its payout ratio (Elton and Gruber [1978])[9]. The converse is true for a below-average tax fund. Furthermore, the *variation* in the *relative* shares held of different equities will be larger the larger (in absolute value) the slope coefficient T_2, that is, the more the fund's target tax rates deviate from the average, in either direction.

It should be noted that though a high bracket portfolio will hold a *relatively* smaller share of low x firms, nonetheless, higher taxes will lead to holding a larger *absolute* share of every stock, including those with low x,[10] (provided their dividend rate is sufficiently low). This behavior reflects, in addition to the variance effect noted earlier, the favorable tax treatment of corporate returns, other than dividends, relative to interest.

3. *Extent of variation in optimum portfolios*

With the help of III.3, one can endeavor to go beyond these qualitative results and get some idea of just how sensitive portfolio composition might be to both taxes and firms' characteristics. One way to measure this sensitivity is to ask how much difference one should expect to find between the largest and smallest relative shares of firms held in the portfolio of funds for which the differences tend to be largest, namely those aimed at extreme tax brackets.

Ignoring inflation at first, and focussing on the role of x, from equation III.3, we obtain:

$$\frac{1}{\bar{n}}\,dn_i = \frac{k(T_2 - \delta_i T_3)}{\bar{n}}\,dx_i = \frac{kT_2(1 - \delta_i)}{\bar{n}}\,dx_i \qquad \text{III.4}$$

since, without inflation, $T_2 = T_3$ as can be seen from III.2b and c.

Thus, the range of variation of n_i depends on the range of x and on the "slope", $T_2(1 - \delta_i)$, which will be largest (in absolute value) when the payout is zero. Consider, then, the lowest possible tax class, $\theta_p = \theta_g = 1$. Relying on the earlier estimate of ⅔ for θ_p, and 0.9 for θ_g, one can put $kT_2/\bar{n}$ at just below 0.3.[11] For a very high tax fund, say, $\theta_p = .5$, $\theta_g = .85$, the slope is even lower, about .25. To estimate the relevant range of x, we can rely on III.3a. If, as seems reasonable, the bulk of the β's can be presumed to fall within the range of 0.5 to 2.0, the range of x is 1.4 to 2.2, or less than one. This implies that the variations in the relative shares of different stocks within one fund, or of the same stock between funds, arising from differences in x, would fall within a quite modest range of about 25 percent.

A second set of questions we can examine with the help of III.3 concerns the impact of leverage on the tax clientele of the firm. First, suppose we classify firms

[9] Elton and Gruber's result is actually stated in terms of the dividend yield, Δ_i/S_i.

[10] From III.2, for the smallest possible value of x which, from III.3a, is θ_g^{-1}, one finds:

$$n_i^m = \frac{\Lambda}{\gamma^m}\left[T_1 + \frac{T_2}{\theta_g} - \frac{T_3}{\theta_g}\delta_i\right] = \frac{\Lambda}{\gamma^m\theta_g}\left[(\theta_g^m)^{-1} - T_3\delta_i\right]$$

which is an increasing function of the tax rate for sufficiently small values of δ_i.

[11] n/k is estimated from II.3 putting $x = 1.5$ (i.e., $\beta = 1$), and $\delta = .5$.

by debt-equity ratios and compute the average tax rate for each leverage class—how should the average tax rate change as leverage increases? In a well known contribution, Kim, Lewellen and McConnell [1979], have provided empirical evidence on this question, based on a large sample of portfolios. They find, as expected, that the estimated average tax rate of stockholders declines systematically as the debt-equity ratio rises from zero to roughly 2 in the highest class, but that the fall—roughly from 41 percent to 37 percent—is modest relative to the implications of Miller's model.

The implication of our model can be deduced from III.3 by examining how the relative share of firms characterized by different leverage differs in the portfolios of funds aimed at different tax brackets. It is apparent from III.2, III.3, and II.13a that leverage affects n_t through, and only through, β^*, and hence x. One can deduce from II.13a that, if leverage were uncorrelated with the asset's β, a rise in d' from zero to 2 would decrease x on the average from 1.5 to 1.2.[12] In light of III.4, this implies a very modest effect of leverage on relative shares. Even for the zero tax fund ($T_2 \simeq .3$) the difference between the relative holdings of unlevered and highly levered firms would be within 10 percent. In reality, there is substantial evidence that leverage is negatively correlated with risk and hence with β. Accordingly, the change in x, and hence n_t, between leverage classes should be even smaller. Clearly, these implications of the model can be easily reconciled with the results of Kim et al., cited above.

A related question concerns the effect that a change in debt policy could be expected to have on the tax clientele of a firm. Here, again, it is found that even for a relatively low β firm, and for a zero tax fund—the combination most sensitive to changes in leverage—the maximum swing in portfolio's relative shares might be within 20 percent, implying that changes in debt policy would not be a source of major shifts in clientele.

Turning finally to the effect of dividend policy, one can readily establish along the lines used above that its effect on portfolio composition should not be very large. But a comparison of the model's implications with empirical evidence is again complicated by the contradictory nature of this evidence. Our model seems to be broadly consistent with the systematic, though modest, association between dividend yield and average tax reported by Lewellen, Stanley, Lease, and Schlarbaum [1978][13] and by Blume, Crockett, and Friend [1974], but not with the rather large differentials that are implied by the results of Elton and Gruber [1970] and Auerbach [1981].[14]

4. *The effect of inflation on portfolio allocation*

How are these various conclusions affected by the presence of inflation? From III.3, one can see that inflation has an effect on n_t through the real after tax rates,

[12] This estimate is obtained by assigning to β in II.13a, the representative value of 1, and noting that the coefficient of d' in II.13a can be put at around ⅔.

[13] At least for tax 1, and dollar weighted positions, as reported in Table 5.

[14] One may suspect that the results of these authors, and others using a similar methodology, may be biassed by the fact that, the higher the dividend yield, the more likely it becomes that institutions that are tax exempt, or not taxed more heavily on dividends then on capital gains, will find it worth while to arbitrage the gains resulting from failure of the ex dividend price to fall commensurately with the dividend.

r_p^m, r_p, and r_c, which appear in T_1, T_2, and T_3, and also through the leverage component of x. The effect through the T coefficients is found to be:

$$\frac{d(n_i/\bar{n})}{dp} = \{-(x_i - \bar{x})[\hat{\theta}_g + (\delta\bar{x})(\hat{\theta}_p\theta_g - \hat{\theta}_g\theta_p)] + [\delta_i x_i - \delta\bar{x})] \cdot [\hat{\theta}_g - \hat{\theta}_p + \bar{x}(\hat{\theta}_p\theta_g - \hat{\theta}_g\theta_p)]\} \frac{1}{\bar{n}^2}\frac{d}{dp}(\hat{r}_p/r_p) \qquad \text{III.5}$$

where $\bar{x}$ and $(\delta\bar{x})$ are values x and δx that correspond to $n = \bar{n}$ and:

$$\frac{d}{dp}(\hat{r}_p/r_p) = \left(\frac{\hat{\theta}_p\tau_p - \theta_p\hat{\tau}_p}{r_p^2}\right)\left(1 - \frac{p}{r}\frac{dr}{dp}\right) \gtreqless 0, \qquad \text{as} \quad \hat{\theta}_p \gtreqless \theta_p \qquad \text{III.6}$$

Conditions III.5 and III.6 are somewhat involved but their broad implications can be readily brought out. First, note that the coefficients of $(x_i - \bar{x})$ and $(\delta_i x_i - \delta\bar{x})$ can, both, be taken to be positive for all investors (or funds). We can then infer that for an *above average tax bracket*, for whom, according to III.6, $\hat{r}_p/r_p$ declines with p, there will be a tendency for inflation to increase the relative share of stock i if x_i is above average and the dividend, $\delta_i x_i$, is below average. Conversely, for a below-average tax bracket, will increase if x_i is below and $\delta_i x_i$ above average. But this means that inflation tends to increase n_i in portfolios where it was above average to begin with, and to reduce it when below average. In short, inflation has the effect of magnifying the difference between the portfolio composition of high and low tax funds.

If one tries to quantify the above effects, one runs again into the difficulty that the value of leverage appears unrealistically sensitive to the rate of inflation, especially if inflation is accompanied by a fall in the after tax real rate, r_p.

Inflation also tends to raise x through the leverage effect, but this increase does not seem to have systematic consequences for portfolio diversity (except through some increase in the range of x).

From these results one can infer that inflation should have an appreciable effect in increasing portfolio diversity between tax brackets. At the same time, the considerations stated in II.5 suggest that the numerical implications of III.3 and III.6 should be heavily discounted on the ground that nonneutral inflation effects should not, and will not, be taken as permanent by investors. As for empirical evidence, none seems presently available on this issue.

5. *Implications of a non-diagonal variance-covariance matrix*

In general, the variance-covariance matrix, $[M]$, will not be diagonal, and equation III.3 has to be replaced by:

$$n_i = k\left[T_1 + \frac{1}{\Lambda}\sum_{j=1}^{N} M_{ij}^{-1}(\mu_j^* - r_c D_j)(T_2 - \delta_j T_3)\right] \qquad \text{III.7}$$

where M_{ij}^{-1} denotes the elements of the inverse.

Thus, n_i now depends not only on characteristics of stock i, but in principle, on those of all other stocks as well. The meaning of the additional terms in III.7 is readily understood. The first order condition requires that, at the margin, the risk premium for every stock held be equal to that stock's contribution to portfolio variance. But, except when $[M]$ is diagonal, raising n_i^m will contribute to variance (positively or negatively) through the covariance with every other stock in the portfolio, as shown by the last term of II.4; $[M]\mathbf{n}^m = n_i\mu_{ii} + \sum_{j\neq 1} n_j\mu_{ij}^*$. The summation in III.7 corresponds precisely to the summation term above, but with n_j expressed in terms of its ultimate determinants, and thus allows for the dependence of each n on every other one.

To see the implications of this generalization, we note that III.7 requires no change in our conclusions about how a stock's own characteristics interact with tax rates in contributing to the attractiveness of that stock for a given tax bracket. As for the effect of all other stocks, we can think of them as falling into two classes: those which have a net positive correlation with, and may be thought of as substitutes for, stock i, and those, more rare, with a negative net correlation, which behave like complements of stock i. The demand for i declines with the attractiveness of its substitutes and rises with that of its complements.

Despite these indirect influences, one would normally expect n_i to be largely determined by the characteristics of stock i. But one cannot reach any definite conclusion unless one imposes restrictions on the variance-covariance matrix, as was done by Elton and Gruber [1978].

The same difficulty arises in analyzing the implications of the generalization III.7 for portfolio diversity. There is, nonetheless, one broad generalization that may be put forward in this case, suggesting a downward bias in our estimates of section III.3, namely that the prevalence of high substitution is likely to increase the extent of portfolio diversity and specialization until, in the limit, perfect correlation leads to the prevalence of corner solutions.[15]

Concluding Remarks and Agenda

In closing, attention must first be called to certain limitations of the current analysis of the demand side, mainly the failure to impose appropriate constraints on portfolios, such as limitations on short sales and on personal debt. This omission may be particularly serious in the case of debt, since it would appropriately give a greater role to wealth in portfolio composition. As one might expect, with such a constraint, it is no longer true that every portfolio includes every stock (in positive or negative amounts). Firms then have a clientele including only a subset of investors and the parameters of equations like II.12 or II.15 might differ between firms reflecting that clientele.

A second limitation is the failure to model explicitly the market for tax exempt securities, but this task is really trivial and the gap has no appreciable effect on the results.

Finally, closure of the model would require combining the demand analysis with an analysis of the forces limiting the supply of corporate debt. This has actually been done in good measure elsewhere (e.g., DeAngelo and Masulis [1980];

[15] This conclusion is illustrated by the result of Elton and Gruber [1978] for the case of equal correlation, equation (B.4); as ρ tends to unity, the fraction invested in stock j tends to plus or minus infinity.

Williamson [1981]). From the point of view of integration with the demand side, we note that the various supply limiting mechanisms which have been analyzed basically imply that an expansion of debt—total assets constant—must eventually either reduce the expected value of the flow, μ, produced by the firm, and/or cause lD to rise at a decreasing rate, because of decreasing probability of utilizing the costly tax shield. If II.12 is modified along the above lines, it turns into a nonlinear function of D_t with negative second derivative, and V will, in general, be maximized for some interior value of D_t, say $\hat{D}_t$. This maximization will fix the individual and aggregate supply of debt.

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